Xi＇an Jiaotong－Liverpool University

| PAPER CODE | EXAMINER | DEPARTMENT | TEL |
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# $1^{\text {st }}$ SEMESTER 2012／13 Final Examination <br> BA ENGLISH－Year 2 

BA ACCOUNTANCY－Year 2
BA ECONOMICS－Year 2

## QUANTITATIVE METHODS FOR ACCOUNTING AND FINANCE

TIME ALLOWED： 2 hours

## INSTRUCTIONS TO CANDIDATES

1，This is a closed－book examination，which is to be written without books，tapes， or notes．

2，Total marks available are 90，divided in Section $A$（40 marks）and B（50 marks）．Answer all questions．

3，In section A， 4 marks will be awarded for each correct answer and 0 marks for each wrong answer provided．There is NO penalty for providing a wrong answer．

4，In section B，a total of 50 marks are available．The number of marks awarded for each question is given in［］after each question．

5，Statistical distribution tables are provided on pages 10 to $\mathbf{1 4}$ of this question paper．

6，Answer should be written in the answer sheet（s）and／or booklet（s）provided． Only English solutions are accepted．

7，The university approved calculator－Casio FS82ES／83ES can be used．
8，All materials must be returned to the exam supervisor upon completion of the exam．Failure to do so will be deemed academic misconduct and will be dealt with accordingly．

## SECTION A（Multiple Choice Questions）

Choose the one alternative that best completes the statement or answers the question．（4 marks per question，Total marks 40）

A1．Suppose a factory produces widgets using an amount of labour，$x$ ．The total cost is given by the following function：

$$
\text { Total Cost }=x^{3}-3 x^{2}-45 x+6
$$

What is the amount of labour that minimizes cost？
a．－3
b． 5
c． 3
d．-5

A2．In hypothesis testing，we make a Type I error if we：
a．Reject the null hypothesis，when the null hypothesis is true．
b．Do not reject the null hypothesis when the null hypothesis is true．
c．Reject the null hypothesis when the null hypothesis is false．
d．Do not reject the null hypothesis，when the null hypothesis is false

A3．Suppose we have the following frequency table．
Table 2 －Data to accompany question A3．

| Marks | Frequency |
| :---: | :---: |
| $50-59$ | 5 |
| $60-69$ | 11 |
| $70-79$ | 15 |
| $80-89$ | 10 |
| $90-100$ | 2 |

What is the upper class boundary of the class interval＇50 to 59 ＇？
a． 59
b． 59.5
c． 59.75
d． 60

A4．Suppose we have the following frequency table．

Table 3 －Data to accompany question A4．

| Classes | Frequency |
| :---: | :---: |
| $50-60$ | 5 |
| $60-70$ | 11 |
| $70-80$ | 15 |
| $80-90$ | 10 |
| $90-100$ | 2 |

What is the third quartile of the data？
a． 80
b． 82
c． 84
d． 85

A5．Suppose you are trying to re－finance a $£ 5,000,000$ loan．Currently your interest payments at the end of the year correspond to $8 \%$ of your total loan．In your new loan interest payments at the end of the year would be reduced to $6 \%$ of your total loan．To get the new loan you must repay your old loan and an extra $£ 500,000$ in fees．How many months will it take for the new lower monthly interest payment re－finance costs？
a． 150
b． 12.5
c． 85.7
d． 7.1

A6．A friend of yours has offered to sell you $50 \%$ of his company．The company has the following cash flows（in thousand RMB）：

Table 4 －Data on costs and revenue over a five（5）years period，to accompany question A6．

| Year | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Costs | 20,000 | 0 | 0 | 5,000 | 5,000 |
| Revenue | 10,000 | 12,000 | 18,000 | 10,000 | 3,000 |

If the discount rate is $7 \%$ ，and you need to pay 30 million RMB for $50 \%$ of the company，what is the Net Present Value（NPV）of this investment（rounded to the second decimal place）？
a．11，782．66（thousand RMB）
b．20，891．33（thousand RMB）
c．$-11,782.66$（thousand RMB）
d．－20，891．33（thousand RMB）

A7．A bag contains 50 balls，of which 18 are red and 32 are yellow．Three balls are drawn separately from the bag．What is the probability that there is at least one yellow ball when each ball drawn is not replaced before the next draw？
a． 0.958
b． 0.918
c． 0.902
d． 0.858

A8．Suppose a variable $Y$ has a normal distribution with a mean of 200 and a standard deviation of 40 ．What is the probability that $Y$ being between 150 and 220 ？
［4 marks］
a． 0.1056
b． 0.3085
c． 0.5859
d． 0.4141

A9．ABC company appraises his new employees using the views of two managers．The two mangers give their ranks for the staff in as follows．

Table 4 －Data on ranking of managers，accompanying
question A9．

| People | A | B | C | D | E | F | G | H | I |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Rank 1 | 3 | 8 | 2 | 4 | 1 | 6 | 9 | 7 | 5 |
| Rank 2 | 1 | 7 | 3 | 5 | 2 | 9 | 4 | 8 | 6 |

What is the Spearman＇s rank correlation coefficient for the two ranking？
a． 0.63
b． 0.48
c．-0.21
d． 0.73

A10．A Consumer Protection Agency is worried about the affordability of health care and is looking at the individual＇s average health expenditure．Average income in the country is $€ 7,500$ and the agency aims to cap average health expenditures at $20 \%$ of average income．They interviewed a random sample of 250 people and found that on average people spend $€ 1,689.30$ per year on health care，with a standard deviation of $€ 800$ ． Are people on average spending more than $20 \%$ of their income on health？What is the value of the statistical test that can answer this question？
a．Yes，people on average are spending more than $20 \%$ of their income on health． The statistical test has a value of 3.741
b．No，people on average are spending less than $20 \%$ of their income on health． The statistical test has a value of 3.741
c．Yes，people on average are spending more than $20 \%$ of their income on health． The statistical test has a value of 0.24
d．No，people on average are spending less than $20 \%$ of their income on health． The statistical test has a value of 0.24

## SECTION B（Short Answer Questions，total marks 50）

B11．Suppose you are thinking of buying a new machine to make widgets in your factory． You can choose two types of machines．You were given the following data on monthly widget production for each of the machines：

Table 5 －Data on units produced by two（2）machines over a period of 12
months，to accompany question B11．

| Month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Machine 1 | 112 | 100 | 95 | 102 | 90 | 96 | 93 | 98 | 101 | 91 | 117 | 108 |
| Machine 2 | 155 | 121 | 104 | 125 | 90 | 107 | 99 | 113 | 124 | 94 | 172 | 144 |

The company is well run and makes $5 \%$ return on capital invested．Machine 1 costs $\$ 200$ to buy and each widget made using this machine costs only $\$ 3$ ．Machine 2 on the other hand costs $\$ 100$ to buy and each widget costs $\$ 5$ to make．Both machines last 5 years，before they completely stop working．Through inquiries，your Market Research Department has found that only 200 people are interested in the widgets if they are free，and 140 people will buy your product at $\$ 10$ a unit．Your Market Research Department recommends that you sell the new widgets at $\$ 6$ ．
a．Is the productivity of machine 1 statistically different from the productivity of machine 2 ？
b．How does the sampling method used by the Market Research Department affect the validity of their findings？What sampling technique would you use？
［7 marks］
c．What price do you need to charge per widget so that you break－even for machine 1？（Make sure you can produce enough to meet demand on an average month）What is the break－even price for machine 2 ？
d．Suppose you sell the new product at the price recommended by the Market Research Department，and produce the average monthly number of widgets． What is the net present value of buying machine 1？What is the net present value for buying machine 2 ？Which machine would you buy？

B12．Tires of a certain brand are tested in a laboratory by running the following experiment．After being mounted on a car，the tires were rotated from position to position every 500 miles，and the groove depth was measured in mils（ 0.001 inches） initially and after every 2000 miles．Measurements were made at six equiangular positions on each of six grooves around the circumference of every tire．The following table gives the averages of the six equiangular measurements on the outside groove of one time after every 2000 miles up to 12,000 miles．

Table 6 －Data on average groove depth and mileage，to accompany question

| Mileage（in 1000 miles） | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Groove Depth（in mils） | 198.5 | 164.3 | 148.3 | 126.8 | 115.1 | 102.4 | 88.6 |

a．Plot a scatter diagram of the data．What does this suggest？
b．Calculate the linear regression equation that best fits the data．
c．Find Pearson＇s coefficient of correlation．
［3 marks］
d．What is your estimate of the average groove depth after the tire was run for 16,000 miles？

B13．Answer the following questions：
a．According to the past data of an insurance company，each driver holding a motor insurance policy in a certain area would expect an average of 0.15 accidents a year．There are totally 200 drivers holding the motor insurance policy．What is the probability that more than 25 drivers will have accidents in one year？
b．In a large city，accidents occur at random at the mean rate of 50 per week． What is the probability that the number of accidents is between 40 and 55 in a given week？

## －－－END OF PAPER－－－

Table 7 －Probabilities for the Normal distribution．Source：Burton，G．，G．Carrol and S．Wall，Quantitative Methods for Business and Economics（2 ${ }^{\text {nd }}$ Ed．）Essex，Prentice Hall， 2002

## Probabilities for the Normal distribution



| $Z$ Score | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| 1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| 1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| 1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| 1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| 1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 ， | 0.0708 | 0.0694 | 0.0681 |
| 1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| 1.6 | 0.0548 | 0.0537 | 0.0562 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| 1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| 1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| 1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| 2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| 2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| 2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| 2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| 2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0072 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| 2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| 2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| 2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| 2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| 2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| 3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |

Table 8 －Cumulative Binomial probabilities．Source：Burton，G．，G．Carrol and S． Wall，Quantitative Methods for Business and Economics（2 ${ }^{\text {nd }}$ Ed．）Essex，Prentice Hall， 2002

## Cumulative Binomial probabilities

|  |  | $p=0.01$ | 0.05 | 0.10 | 0.20 | 0.30 | 0.40 | 0.45 | 0.50 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $n=5$ | $r=0$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
|  | 1 | 0.0490 | 0.2262 | 0.4095 | 0.6723 | 0.8319 | 0.9222 | 0.9497 | 0.9688 |
|  | 2 | 0.0010 | 0.0226 | 0.0815 | 0.2627 | 0.4718 | 0.6630 | 0.7438 | 0.8125 |
|  | 3 |  | 0.0012 | 0.0086 | 0.0579 | 0.1631 | 0.3174 | 0.4069 | 0.5000 |
|  | 4 |  |  | 0.0005 | 0.0067 | 0.0308 | 0.0870 | 0.1312 | 0.1875 |
|  | 5 |  |  |  | 0.0003 | 0.0024 | 0.0102 | 0.0185 | 0.0313 |
| $n=10$ | $r=0$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
|  | 1 | 0.0956 | 0.4013 | 0.6513 | 0.8926 | 0.9718 | 0.9940 | 0.9975 | 0.9990 |
|  | 2 | 0.0043 | 0.0861 | 0.2639 | 0.6242 | 0.8507 | 0.9536 | 0.9767 | 0.9893 |
|  | 3 | 0.0001 | 0.0115 | 0.0702 | 0.3222 | 0.6172 | 0.8327 | 0.9004 | 0.9453 |
|  | 4 |  | 0.0010 | 0.0128 | 0.1209 | 0.3504 | 0.6177 | 0.7430 | 0.8281 |
|  | 5 |  | 0.0001 | 0.0016 | 0.0328 | 0.1503 | 0.3669 | 0.4956 | 0.6230 |
|  | 6 |  |  | 0.0001 | 0.0064 | 0.0473 | 0.1662 | 0.2616 | 0.3770 |
|  | 7 |  |  |  | 0.0009 | 0.0106 | 0.0548 | 0.1020 | 0.1719 |
|  | 8 |  |  |  | 0.0001 | 0.0016 | 0.0123 | 0.0274 | 0.0547 |
|  | 9 |  |  |  |  | 0.0001 | 0.0017 | 0.0045 | 0.0107 |
|  | 10 |  |  |  |  |  | 0.0001 | 0.0003 | 0.0010 |

where
$p$ is the probability of a characteristic（e．g．a defective item），
$n$ is the sample size and
$r$ is the number with that characteristic．
Note：All probabilities are for＇$r$ or more successes＇．Only selected values for $n$ and $r$ are shown in this table．

Table 9 －Cumulative Poisson probabilities．Source：Burton，G．，G．Carrol and S． Wall，Quantitative Methods for Business and Economics（2 ${ }^{\text {nd }}$ Ed．）Essex，Prentice Hall， 2002

## Cumulative Poisson probabilities

|  | $\mu=1.0$ | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r=$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
|  | 0.6321 | 0.8647 | 0.9502 | 0.9817 | 0.9933 | 0.9975 | 0.9991 |
|  | 0.2642 | 0.5940 | 0.8009 | 0.9084 | 0.9596 | 0.9826 | 0.9927 |
|  | 0.0803 | 0.3233 | 0.5768 | 0.7619 | 0.8753 | 0.9380 | 0.9704 |
|  | 0.0190 | 0.1429 | 0.3528 | 0.5665 | 0.7350 | 0.8488 | 0.9182 |
|  | 0.0037 | 0.0527 | 0.1847 | 0.3712 | 0.5595 | 0.7149 | 0.8270 |
|  | 0.0006 | 0.0166 | 0.0839 | 0.2149 | 0.3840 | 0.5543 | 0.6993 |
|  | 0.0001 | 0.0011 | 0.0335 | 0.1107 | 0.2378 | 0.3937 | 0.5503 |
|  |  | 0.0002 | 0.0119 | 0.0511 | 0.1334 | 0.2560 | 0.4013 |
|  |  |  | 0.0038 | 0.0214 | 0.0681 | 0.1528 | 0.2709 |
| 10 |  |  | 0.0011 | 0.0081 | 0.0318 | 0.0839 | 0.1695 |
| 11 |  |  | 0.0003 | 0.0028 | 0.0137 | 0.0426 | 0.0985 |
| 12 |  |  | 0.0001 | 0.0009 | 0.0055 | 0.0201 | 0.0534 |
| 13 |  |  |  | 0.0003 | 0.0020 | 0.0088 | 0.0270 |
| 14 |  |  |  | 0.0001 | 0.0007 | 0.0036 | 0.0128 |
| 15 |  |  |  |  | 0.0002 | 0.0014 | 0.0057 |
| 16 |  |  |  |  | 0.0001 | 0.0005 | 0.0024 |
| 17 |  |  |  |  |  | 0.0002 | 0.0010 |
| 18 |  |  |  |  |  | 0.0001 | 0.0004 |
| 19 |  |  |  |  |  |  | 0.0001 |

where
$\mu(=n p)_{s}$ is the average number of times a characteristic occurs and $r$ is the number of occurrences．
Note：All probabilities are for＇r or more successes＇．

Table 10 －Student t distribution critical values．Source：Burton，G．，G．Carrol and S． Wall，Quantitative Methods for Business and Economics（2 ${ }^{\text {nd }}$ Ed．）Essex，Prentice Hall， 2002

## Student $t$ Critical values



| Probability | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $v=1$ | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 120 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 |
| $\infty$ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |

where $v$ is the number of degrees of freedom．

Table 11 －Chi－Square distribution critical values．Source：Burton，G．，G．Carrol and S．Wall，Quantitative Methods for Business and Economics（2 ${ }^{\text {nd }}$ Ed．）Essex，Prentice Hall， 2002

## $\chi^{2}$ Critical values



| Probability <br> v | 0.250 | 0.100 | 0.050 | 0.025 | 0.010 | 0.005 | 0.001 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.32 | 2.71 | 3.84 | 5.02 | 6.63 | 7.88 | 10.8 |
| 2 | 2.77 | 4.61 | 5.99 | 7.38 | 9.21 | 10.6 | 13.8 |
| 3 | 4.11 | 6.25 | 7.81 | 9.35 | 11.3 | 12.8 | 16.3 |
| 4 | 5.39 | 7.78 | 9.49 | 11.1 | 13.3 | 14.9 | 18.5 |
| 5 | 6.63 | 9.24 | 11.1 | 12.8 | 15.1 | 16.7 | 20.5 |
| 6 | 7.84 | 10.6 | 12.6 | 14.4 | 16.8 | 18.5 | 22.5 |
| 7 | 9.04 | 12.0 | 14.1 | 16.0 | 18.5 | 20.3 | 24.3 |
| 8 | 10.2 | 13.4 | 15.5 | 17.5 | 20.3 | 22.0 | 26.1 |
| 9 | 11.4 | 14.7 | 16.9 | 19.0 | 21.7 | 23.6 | 27.9 |
| 10 | 12.5 | 16.0 | 18.3 | 20.5 | 23.2 | 25.2 | 29.6 |
| 11 | 13.7 | 17.3 | 19.7 | 21.9 | 24.7 | 26.8 | 31.3 |
| 12 | 14.8 | 18.5 | 21.0 | 23.3 | 26.2 | 28.3 | 32.9 |
| 13 | 16.0 | 19.8 | 22.4 | 24.7 | 27.7 | 29.8 | 34.5 |
| 14 | 17.1 | 21.1 | 23.7 | 26.1 | 29.1 | 31.3 | 36.1 |
| 15 | 18.2 | 22.3 | 25.0 | 27.5 | 30.6 | 32.8 | 37.7 |
| 16 | 19.4 | 23.5 | 26.3 | 28.8 | 32.0 | 34.3 | 39.3 |
| 17 | 20.5 | 24.8 | 27.6 | 30.2 | 33.4 | 35.7 | 40.8 |
| 18 | 21.6 | 26.0 | 28.9 | 31.5 | 34.8 | 37.2 | 42.3 |
| 19 | 22.7 | 27.2 | 30.1 | 32.9 | 36.2 | 38.6 | 43.8 |
| 20 | 23.8 | 28.4 | 31.4 | 34.2 | 37.6 | 40.0 | 45.3 |
| 21 | 24.9 | 29.6 | 32.7 | 35.5 | 38.9 | 41.4 | 46.8 |
| 22 | 26.0 | 30.8 | 33.9 | 36.8 | 40.3 | 42.8 | 48.3 |
| 23 | 27.1 | 32.0 | 35.2 | 38.1 | 41.6 | 44.2 | 49.7 |
| 24 | 28.2 | 33.2 | 36.4 | 39.4 | 43.0 | 45.6 | 51.2 |
| 25 | 29.3 | 34.4 | 37.7 | 40.6 | 44.3 | 46.9 | 52.6 |
| 26 | 30.4 | 35.6 | 38.9 | 41.9 | 45.6 | 48.3 | 54.1 |
| 27 | 31.5 | 36.7 | 40.1 | 43.2 | 47.0 | 49.6 | 55.5 |
| 28 | 32.6 | 37.9 | 41.3 | 44.5 | 48.3 | 51.0 | 56.9 |
| 29 | 33.7 | 39.1 | 42.6 | 45.7 | 49.6 | 52.3 | 58.3 |
| 30 | 34.8 | 40.3 | 43.8 | 47.0 | 50.9 |  |  |
| 40 | 45.6 | 51.8 | 55.8 | 59.3 | 63.7 | 66.8 | 73.4 |
| 50 | 56.3 | 63.2 | 67.5 | 71.4 | 76.2 | 79.5 | 86.7 |
| 60 | 67.0 | 74.4 | 79.1 | 83.3 | 88.4 | 92.0 | 99.6 |
| 70 | 77.6 | 85.5 | 90.5 | 95.0 | 100 | 104 | 112 |
| 80 | 88.1 | 96.6 | 102 | 107 | 112 | 116 | 125 |
| 90 | 98.6 | 108 | 113 | 118 | 124 | 128 | 137 |
| 100 | 109 | 118 | 124 | 130 | 136 | 140 | 149 |

where $v$ is the number of degrees of freedom．

