| PAPER CODE | EXAMINER | DEPARTMENT | TEL |
| :---: | :---: | :---: | :---: |
| ECO111 | Tiago Freire | IBSS | $\mathbf{0 4 5 0}$ |

# $1^{\text {st }}$ SEMESTER 2013／14 Final Examination <br> BA ACCOUNTANCY－Year 2 <br> BA BUSINESS ADMINISTRATION－Year 2 <br> QUANTITATIVE METHODS 

TIME ALLOWED： 2 hours

## INSTRUCTIONS TO CANDIDATES

1，This is a closed－book examination，which is to be written without books，tapes， or notes．

2．Total marks available are 100，divided in Section A（40 marks）and B（60 marks）．Answer all questions．

3，In section A， 4 marks will be awarded for each correct answer and $\mathbf{0}$ marks for each wrong answer provided．There is NO penalty for providing a wrong answer．

4，In section B，a total of $\mathbf{6 0}$ marks are available．The number of marks awarded for each question is given in［］after each question．

5，Statistical distribution tables are provided on pages 10 to 14 of this question paper．

6，Answer should be written in the answer sheet（s）and／or booklet（s）provided． Only English solutions are accepted．

7，The university approved calculator－Casio FS82ES／83ES can be used．
8，All materials must be returned to the exam supervisor upon completion of the exam．Failure to do so will be deemed academic misconduct and will be dealt with accordingly．

## SECTION A（Multiple Choice Questions）

Choose the one alternative that best completes the statement or answers the question．（4 marks per question，Total marks 40）

A1．A monopolist has the following demand function：

$$
p=20-0.5 q
$$

And her cost function is：

$$
C=0.04 q^{3}-1.94 q^{2}+32.96 q-1.44
$$

Where $C$ is total cost，$q$ is quantity and $p$ is price．What is the monopolist＇s profit？
a．$\quad 1.44$
b． 6
c． 18
d． 17

A2．How much must I invest at a rate of $6 \%$ payable annually to obtain $£ 5,000$ after 4 years？
a．$£ 2,855.34$
b．$£ 3,960.47$
c．$£ 4,706.67$
d．$£ 5,000$

A3．A property investment company has the opportunity to buy an office building now for $£ 300,000$ ．It will cost $£ 100,000$ to refurbish it，which will be payable at the end of year 1 ．The company expects to be able to sell it for $£ 500,000$ in 3 years’ time．What is the net present value of the project，assuming a discount rate of $6 \%$ ？
a．$£ 35,210.60$
b．$£ 19,890.50$
c．$£ 25,470.02$
d．$£ 50,000$

A4．The number of passengers on a bus route was recorded over a period of time，to give the following data：

Table 1 －Data to accompany question A4．

| Number of <br> Passengers | Frequency |
| :---: | :---: |
| $0 \leq n<10$ | 3 |
| $10 \leq n<20$ | 7 |
| $20 \leq n<30$ | 12 |
| $30 \leq n<40$ | 13 |
| $40 \leq n<50$ | 29 |
| $50 \leq n<60$ | 27 |

What is the inter－quartile range（to the nearest integer）？
a． 31
b． 18
c． 28
d． 21

A5．Data in the 30 largest bond funds provided one－year and five－year percentage returns for the period ending the $31^{\text {st }}$ of March， 2000 （The Wall Street Journal， $10^{\text {th }}$ of April， 2000）．Suppose we consider a one－year return in excess of $2 \%$ to be high and a five－ year return in excess of $44 \%$ to be high．One－half of the funds had a one－year return in excess of $2 \%, 12$ of the funds had a five－year return in excess of $44 \%$ ，and six of the funds had both a one－year return in excess of $2 \%$ and a five year return in excess of 44\％．

What is the probability that a fund had neither a high one－year return nor a high five－ year return？
a． 0.3
b． 0.7
c． 0.1
d． 0.9

A6．There is a $90 \%$ probability that each of 10 jury members casts a vote of not guilty． What is the probability that 7 or less members vote not guilty？Assume that each jury member＇s vote is independent of the others．
a． 0.0001
b． 0.0702
c． 0.0815
d． 0.0043

A7．Annually the U．S．Census Bureau provides estimates of a variety of demographic characteristics of the U．S．population．In terms of categorizing people by race，the bureau divides the population into those of＂Hispanic＂origin and those not． Within these two categories，a person can be associated with one of several races． For 2007，the bureau gives the following estimates：

Table 2 －Data to accompany question A6．

|  | Hispanic | Not Hispanic |
| :---: | :---: | :---: |
| Asian | 0.001 | 0.043 |
| Black | 0.006 | 0.123 |
| White | 0.139 | 0.660 |
| Other | 0.005 | 0.023 |

What is the probability that a person is white，given that that you know that the person is of Hispanic origin？
a． 0.123
b． 0.660
c． 0.920
d． 0.174

A8．According to a survey，salaries of dentists have an average of $£ 48,000$ with a standard deviation of $£ 3,500$ ．If the salary of a dentist is normally distributed what salary is such that $20 \%$ of dentists have a higher salary？
a．$£ 60,566$
b．$£ 52,552$
c．$£ 48,213$
d．$£ 50,946$

A9．Management claim that the bonuses paid to workers at a plant average $£ 1,000$ ．A random sample of 100 workers gives an average bonus of only $£ 975$ and a standard deviation of $£ 100$ ．Are workers on average being paid $£ 1000$ in bonuses？What is the value of the statistical test that can answer this question？Assume a $5 \%$ significance level．
a．Yes，workers are on average being paid $£ 1000$ in bonuses．The statistical test has a value of -2.5
b．No，workers are on average not being paid $£ 1000$ in bonuses．The statistical test has a value of－2．5
c．Yes，workers are on average being paid $£ 1000$ in bonuses．The statistical test has a value of -0.25
d．No，workers are on average not being paid $£ 1000$ in bonuses．The statistical test has a value of -0.25

A10．In hypothesis testing，for a given level of significance，alpha，you can decrease the chance of Type II error by：
a．Increasing the sample size
b．Decreasing the sample size
c．Increasing the skewness
d．Decreasing the skewness

## SECTION B（Short Answer Questions，total marks 60）

B11．Suppose ideally，a worker seeking a new job in a particular industry should acquire information about wage rates offered by all firms in the industry．However，this information search could be time consuming and costly．In particular，the longer an unemployed worker searches for a higher wage，the greater will be the loss in income． Therefore，workers may not find it worthwhile to search until they find the highest available wage rate．The result is that managers may not have to pay top dollar to attract workers．These factors help explain the existing disparity in wage rates among firms．Suppose the distribution of wage rates nationwide that would be offered to a particular skilled worker can be approximated by a normal distribution with $\mu=$ $\$ 10.50$ per hour and $\sigma=\$ 1.25$ per hour．Suppose the worker were to undertake a nationwide job search．
a．What proportion of the wage rates that would be offered to the worker would be greater than $\$ 12.00$ per hour？
b．If the worker randomly selects one of the many jobs offers received，what is the probability that the wage rate will be between $\$ 11.00$ and $\$ 12.00$ ？
［4 marks］
c．If the worker randomly selects one of the many jobs offers received，what is the probability that the wage rate will be between $\$ 9.5$ and $\$ 12.00$ ？
［4 marks］
d．If the worker tries to avoid being in the $25 \%$ of the lowest paid category，what is the minimum wage rate that he will accept？

B12．A home products firm has developed a new solar energy heater and now must decide whether to market it．The success of the product，as measured by its profit contribution，depends on the proportion of households in the target market that will purchase it．If the demand is high（that is if there is a favorable market for the heater）， the firm could realize a profit of $\$ 900,000$ ．If the market is not favorable，the firm could lose $\$ 500,000$ ．Of course，the firm doesn＇t have to market the heater，in which case there is no cost．Based on experience，the marketing manager believes that the probability of a favorable market is 0.55 and the probability of an unfavorable market is 0.45 ．

The firm may hire Home Product Promotions Inc．（HPP），a marketing research firm， to conduct a study of the market，at a fee of $\$ 5,000$ ．Past studies by HPP show the following conditional probabilities：

Probability of favorable study given a favorable market $=0.80$
Probability of unfavorable study given an unfavorable market $=0.90$
a．List the conditional probabilities and compute the joint probabilities．What are the probabilities of the different predictions by HPP（keep 3 decimal digits）？
［2 marks］
b．Compute the posterior or revised probabilities using the information given above．
c．Draw a decision tree．
d．Should the firm hire HPP to conduct the market research or not？Explain your reasoning．
e．How does the sampling method used in the market study affect your answer in the previous question？

B13．The number of staff working in a pub during an evening shift and the number of abusive customers encountered that evening for 5 consecutive Monday evenings are recorded in the table bellow．

Table 3 －Data to accompany question B13．

| Number of <br> staff | Abusive <br> customers |
| :---: | :---: |
| 3 | 10 |
| 4 | 11 |
| 2 | 13 |
| 6 | 15 |
| 2 | 10 |

a．Plot a scatter diagram of the data．What does it suggest？
b．Calculate the linear regression equation that best fits the data．
c．Comment on how useful the number of staff is in explaining the number of abusive customers．What can you say about causality？

## Probabilities for the Normal distribution



| $z$ Score | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| 1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| 1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| 1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| 1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| 1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 ， | 0.0708 | 0.0694 | 0.0681 |
| 1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| 1.6 | 0.0548 | 0.0537 | 0.0562 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| 1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| 1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| 1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| 2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| 2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| 2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| 2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| 2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0072 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| 2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| 2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| 2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| 2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| 2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| 3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |

## Cumulative Binomial probabilities

|  |  | $p=0.01$ | 0.05 | 0.10 | 0.20 | 0.30 | 0.40 | 0.45 | 0.50 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $n=5$ | $r=0$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
|  | 1 | 0.0490 | 0.2262 | 0.4095 | 0.6723 | 0.8319 | 0.9222 | 0.9497 | 0.9688 |
|  | 2 | 0.0010 | 0.0226 | 0.0815 | 0.2627 | 0.4718 | 0.6630 | 0.7438 | 0.8125 |
|  | 3 |  | 0.0012 | 0.0086 | 0.0579 | 0.1631 | 0.3174 | 0.4069 | 0.5000 |
|  | 4 |  |  | 0.0005 | 0.0067 | 0.0308 | 0.0870 | 0.1312 | 0.1875 |
|  | 5 |  |  |  | 0.0003 | 0.0024 | 0.0102 | 0.0185 | 0.0313 |
| $n=10$ | $r=0$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
|  | 1 | 0.0956 | 0.4013 | 0.6513 | 0.8926 | 0.9718 | 0.9940 | 0.9975 | 0.9990 |
|  | 2 | 0.0043 | 0.0861 | 0.2639 | 0.6242 | 0.8507 | 0.9536 | 0.9767 | 0.9893 |
|  | 3 | 0.0001 | 0.0115 | 0.0702 | 0.3222 | 0.6172 | 0.8327 | 0.9004 | 0.9453 |
|  | 4 |  | 0.0010 | 0.0128 | 0.1209 | 0.3504 | 0.6177 | 0.7430 | 0.8281 |
|  | 5 |  | 0.0001 | 0.0016 | 0.0328 | 0.1503 | 0.3669 | 0.4956 | 0.6230 |
|  | 6 |  |  | 0.0001 | 0.0064 | 0.0473 | 0.1662 | 0.2616 | 0.3770 |
|  | 7 |  |  |  | 0.0009 | 0.0106 | 0.0548 | 0.1020 | 0.1719 |
|  | 8 |  |  |  | 0.0001 | 0.0016 | 0.0123 | 0.0274 | 0.0547 |
|  | 9 |  |  |  |  | 0.0001 | 0.0017 | 0.0045 | 0.0107 |
|  | 10 |  |  |  |  |  | 0.0001 | 0.0003 | 0.0010 |

where
$p$ is the probability of a characteristic（e．g．a defective item），
$n$ is the sample size and
$r$ is the number with that characteristic．
Note：All probabilities are for＇$r$ or more successes＇．Only selected values for $n$ and $r$ are shown in this table．

## Cumulative Poisson probabilities



## Student $\boldsymbol{t}$ Critical values



| Probability | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $v=1$ | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 120 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 |
| $\infty$ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |

[^0]
## $\chi^{2}$ Critical values



| Probability <br> v | 0.250 | 0.100 | 0.050 | 0.025 | 0.010 | 0.005 | 0.001 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.32 | 2.71 | 3.84 | 5.02 | 6.63 | 7.88 | 10.8 |
| 2 | 2.77 | 4.61 | 5.99 | 7.38 | 9.21 | 10.6 | 13.8 |
| 3 | 4.11 | 6.25 | 7.81 | 9.35 | 11.3 | 12.8 | 16.3 |
| 4 | 5.39 | 7.78 | 9.49 | 11.1 | 13.3 | 14.9 | 18.5 |
| 5 | 6.63 | 9.24 | 11.1 | 12.8 | 15.1 | 16.7 | 20.5 |
| 6 | 7.84 | 10.6 | 12.6 | 14.4 | 16.8 | 18.5 | 22.5 |
| 7 | 9.04 | 12.0 | 14.1 | 16.0 | 18.5 | 20.3 | 24.3 |
| 8 | 10.2 | 13.4 | 15.5 | 17.5 | 20.3 | 22.0 | 26.1 |
| 9 | 11.4 | 14.7 | 16.9 | 19.0 | 21.7 | 23.6 | 27.9 |
| 10 | 12.5 | 16.0 | 18.3 | 20.5 | 23.2 | 25.2 | 29.6 |
| 11 | 13.7 | 17.3 | 19.7 | 21.9 | 24.7 | 26.8 | 31.3 |
| 12 | 14.8 | 18.5 | 21.0 | 23.3 | 26.2 | 28.3 | 32.9 |
| 13 | 16.0 | 19.8 | 22.4 | 24.7 | 27.7 | 29.8 | 34.5 |
| 14 | 17.1 | 21.1 | 23.7 | 26.1 | 29.1 | 31.3 | 36.1 |
| 15 | 18.2 | 22.3 | 25.0 | 27.5 | 30.6 | 32.8 | 37.7 |
| 16 | 19.4 | 23.5 | 26.3 | 28.8 | 32.0 | 34.3 | 39.3 |
| 17 | 20.5 | 24.8 | 27.6 | 30.2 | 33.4 | 35.7 | 40.8 |
| 18 | 21.6 | 26.0 | 28.9 | 31.5 | 34.8 | 37.2 | 42.3 |
| 19 | 22.7 | 27.2 | 30.1 | 32.9 | 36.2 | 38.6 | 43.8 |
| 20 | 23.8 | 28.4 | 31.4 | 34.2 | 37.6 | 40.0 | 45.3 |
| 21 | 24.9 | 29.6 | 32.7 | 35.5 | 38.9 | 41.4 | 46.8 |
| 22 | 26.0 | 30.8 | 33.9 | 36.8 | 40.3 | 42.8 | 48.3 |
| 23 | 27.1 | 32.0 | 35.2 | 38.1 | 41.6 | 44.2 | 49.7 |
| 24 | 28.2 | 33.2 | 36.4 | 39.4 | 43.0 | 45.6 | 51.2 |
| 25 | 29.3 | 34.4 | 37.7 | 40.6 | 44.3 | 46.9 | 52.6 |
| 26 | 30.4 | 35.6 | 38.9 | 41.9 | 45.6 | 48.3 | 54.1 |
| 27 | 31.5 | 36.7 | 40.1 | 43.2 | 47.0 | 49.6 | 55.5 |
| 28 | 32.6 | 37.9 | 41.3 | 44.5 | 48.3 | 51.0 | 56.9 |
| 29 | 33.7 | 39.1 | 42.6 | 45.7 | 49.6 | 52.3 | 58.3 |
| 30 | 34.8 | 40.3 | 43.8 | 47.0 | 50.9 | 53.7 | 59.7 |
| 40 | 45.6 | 51.8 | 55.8 | 59.3 | 63.7 | 66.8 | 73.4 |
| 50 | 56.3 | 63.2 | 67.5 | 71.4 | 76.2 | 79.5 | 86.7 |
| 60 | 67.0 | 74.4 | 79.1 | 83.3 | 88.4 | 92.0 | 99.6 |
| 70 | 77.6 | 85.5 | 90.5 | 95.0 | 100 | 104 | 112 |
| 80 | 88.1 | 96.6 | 102 | 107 | 112 | 116 | 125 |
| 90 | 98.6 | 108 | 113 | 118 | 124 | 128 | 137 |
| 100 | 109 | 118 | 124 | 130 | 136 | 140 | 149 |

where $v$ is the number of degrees of freedom．


[^0]:    where $v$ is the number of degrees of freedom．

