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PROBABILITY DISTRIBUTION — Extra Exercises

These exercises are here to help you learn how to use the formulas you have seen in the lectures. Please do these exercises before doing your tutorial questions.

Problem 1. Binomial (I):

Consider a binomial experiment with n= 10 and p =0.10. Use the binomial tables to answer the following questions:

(a) Find f(0).
(b) Find f(2).
(c) Find $P(x \leq 2)$
(d) Find $P(x \geq 1)$.
(e) Find $E(x)$
(f) Find $Var(x)$ and σ

Problem 2. Binomial (II):

A Harris interactive survey for InterContinental Hotels & Resorts asked respondents, "When traveling internationally, do you generally venture out on your own to experience culture or stick with your tour group and itineraries?". The survey found that 23% of the respondents stick with their tour group (*USA Today*, January 21, 2004)

(a) In a sample of five international travellers, what is the probability that two will stick with their tour group?
(b) In a sample of five international travellers, what is the probability that at least two will stick with their tour group?

(c) In a sample of ten international travellers, what is the probability that none will stick with their tour group?

Problem 3. Binomial (III):

When a machine is functioning properly, only 3% of the items produced are defective. Assume that we will randomly select two parts produced on the machine and that we are interested in the number of defective pats found.

(a) Describe the conditions under which this situation would be a binomial experiment.
(b) How many experimental outcomes yield one defect?
(c) Compute the probabilities associated with finding no defects, 1 defect, and 2 defects.

Problem 4. Binomial (IV):

Military radar and missile detection systems are designed to warn a country of enemy attacks and issue the warning. Assume that a particular detection system has a 0.90 probability of detecting a missile attack. Answer the following questions using the binomial probability distribution:

(a) What is the probability that 1 detection system will detect an attack?
(b) If 2 detection systems are installed in the same area and operate independently, what is the probability that at least 1 of the systems will detect the attack?
(c) If 3 systems are installed, what is the probability, what is the probability that at least 1 of the systems will detect the attack?
(d) Would you recommend multiple detection systems be operated? Explain.

Problem 5. Binomial (V):

(a) A random variable, X, comes from a binomial distribution with n=50 and p=0.4. What distribution does X follow approximately?
(b) 30% of computer analysts who are hired by Techtronics have programming experience. If a random sample of 35 analysts are selected, what is the approximate probability distribution of the number with programming experience?

Problem 6. Binomial (VI):

The probability that a share price rises from one day to the next is 0.4. A portfolio contains 80 different shares. What assumptions are necessary for the number of shares that have prices that rise on a particular day to be a binomial random variable? What is the mean and variance of the number of share prices that rise on a given day?

Problem 7. Poisson (I):

Consider a Poisson probability distribution with 2 as the average occurrences per time period.

(a) Write the appropriate Poisson probability function?

(b) What is the average number of occurrences in three time periods?

(c) Write the appropriate Poisson probability function to determine the probability of x occurrences in three time periods.

(d) Find the probability of 2 occurrences in one time period.

(e) Find the probability of 6 occurrences in three time periods.

(f) Find the probability of 5 occurrences in two time periods.

Problem 8. Poisson (II):

More than 50 million guests stayed at bed and breakfast (B&B) last year. The Web site for the Bed and Breakfast Inns of North America (http://www,bestinns.com), which averages approximately seven visitors per minute, enables many B&Bs to attract guests without waiting years to be mentioned in guidebooks (*Times*, September 2001).

(a) What is the probability of no Web site visitor in a 1-minute period?

(b) What is the probability of two or more Web site visitors in a 1-minute period?

(c) What is the probability of one or more Web site visitors in a 30-second period?

(d) What is the probability of one or more Web site visitors in a 1-minute period?

Problem 9. Poisson (III):

Airline passengers arrive randomly and independently at the passenger screening facility at a major international airport. The mean arrival rate is 10 passengers per minute.

(a) What is the probability of no arrivals in a 1-minute period?
(b) What is the probability of 3 or fewer arrivals in a 1-minute period?
(c) What is the probability of no arrivals in a 15-second period?
(d) What is the probability of at least 1 arrival in a 15-second period?

Problem 10. Poisson (IV):

A photographic manufacturer produces the same number of lenses (over 5000) each week. On average 3.5 of these are found to be defective. Calculate the approximate probability that:

(a) No defective lenses are produced in a week.
(b) Less than 3 defective lens are produced in a week.

Problem 11. Poisson (V):

A large hotel knows that, on average, 1% of its customers require a special diet for medical reasons. It is hosting a conference for 400 people.

(a) Which probability distribution would you suggest for calculating the exact probability that no customer at the conference will require a special diet? Calculate this probability.
(b) Which probability distribution do you suggest as an approximation to this and why. Calculate an approximate probability that no customers require a special diet.
(c) Compare the two previous answers.
(d) From past records the hotel knows that 0.1% of its customers will require medical attention while staying in the hotel. Calculate the exact and approximate probability that no customers out of the 400 will require medical attention while attending the conference. Is the approximation better or worse than the approximation used in (2)? Why?

Problem 12. Normal Distribution (I):

X is a normal random variable with mean $\mu = 5$ and variance $\sigma^2 = 4$. Calculate the following probabilities:

(a) $P(X > 5.7)$
(b) $P(X < 3.4)$

(c) $P(2.8 < X < 5.1)$
(d) $P(5.7 < X < 6.8)$

Problem 13. Normal Distribution (II):

X is a normal random variable with mean 10 and variance 9. Find a such that:

(a) the probability that X is less than a is 0.51
(b) $P(X > a) = 0.6$
(c) $P(X \geq a) = 0.05$
(d) $P(10 < X < a) = 0.05$

Problem 14. Normal Distribution (III):

The yearly cost of dental claims for the employees of Notooth International is normally distributed with mean $\mu = £75$ and standard deviation $\sigma = £25$.

(a) What proportion of employees can be expected to claim over £120 in a year?
(b) What yearly cost do 30% of employees claims less than?

Problem 15. Normal Distribution (IV):

Petrol consumption for all types of small car is normally distributed with $\mu = 30.5$ m.p.g. (miles per gallon) and $\sigma = 4.5$ m.p.g.. A manufacturer wants to make a car that is more economical than 95% of small cars. What must be its m.p.g.?

Problem 16. Normal Distribution (V):

General Hospital's patient account division has compiled data on the age of accounts receivable. The data collected indicate that the age of accounts follows a normal distribution with $\mu = 28$ days and $\sigma = 8$ days.

(a) What portion of the accounts is between 20 and 40 days old - that is, $P(20 \leq X \leq 40)$?

(b) The hospital administrator is interested in sending reminder letters to the oldest 15% of accounts. How many days old should an account be before a reminder letter is sent?

(c) The hospital administrator wants to give a discount to those accounts that pay their balance by the 21st day. What percentage of the accounts will receive the discount?

Problem 17. Normal Distribution (VI):

The Webster National Bank is reviewing its service charges and interest-paying policies on checking accounts. The average daily balance on personal checking accounts is $550, with a standard deviation of $150. In addition, the average daily balances are normally distributed.

(a) What percentage of personal checking account customers carry average daily balances in excess of $800?

(b) What percentage carry average daily balances below $200?

(c) What percentage carry average daily balances between $300 and $700?

(d) The bank is considering paying interest to customers carrying average daily balances in excess of a certain amount. If the bank does not want to pay interest to more than 5% of its customers, what is the minimum average daily balance it should be willing to pay interest on?

Problem 18. Normal Distribution (VII):

A machine fills containers with a particular product. The standard deviation of filing weights computed from past data is 0.6 ounces. If only 2% of the containers hold less than 18 ounces, what is the mean filling weight for the machine? That is, what must μ equal? Assume that filing weights have a normal distribution.

Problem 19. Normal Distribution (VIII):

A machine makes a product, with 5% of units having faults. In a sample of 20 units, what is the probability that at least 1 is defective? In a sample of 200 units, what is the probability that at least 10 are defective?